BISHOP SCOTT BOYS' SCHOOL

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STUDY COURSE MATERIAL

MATHEMATICS

SESSION-2020-21

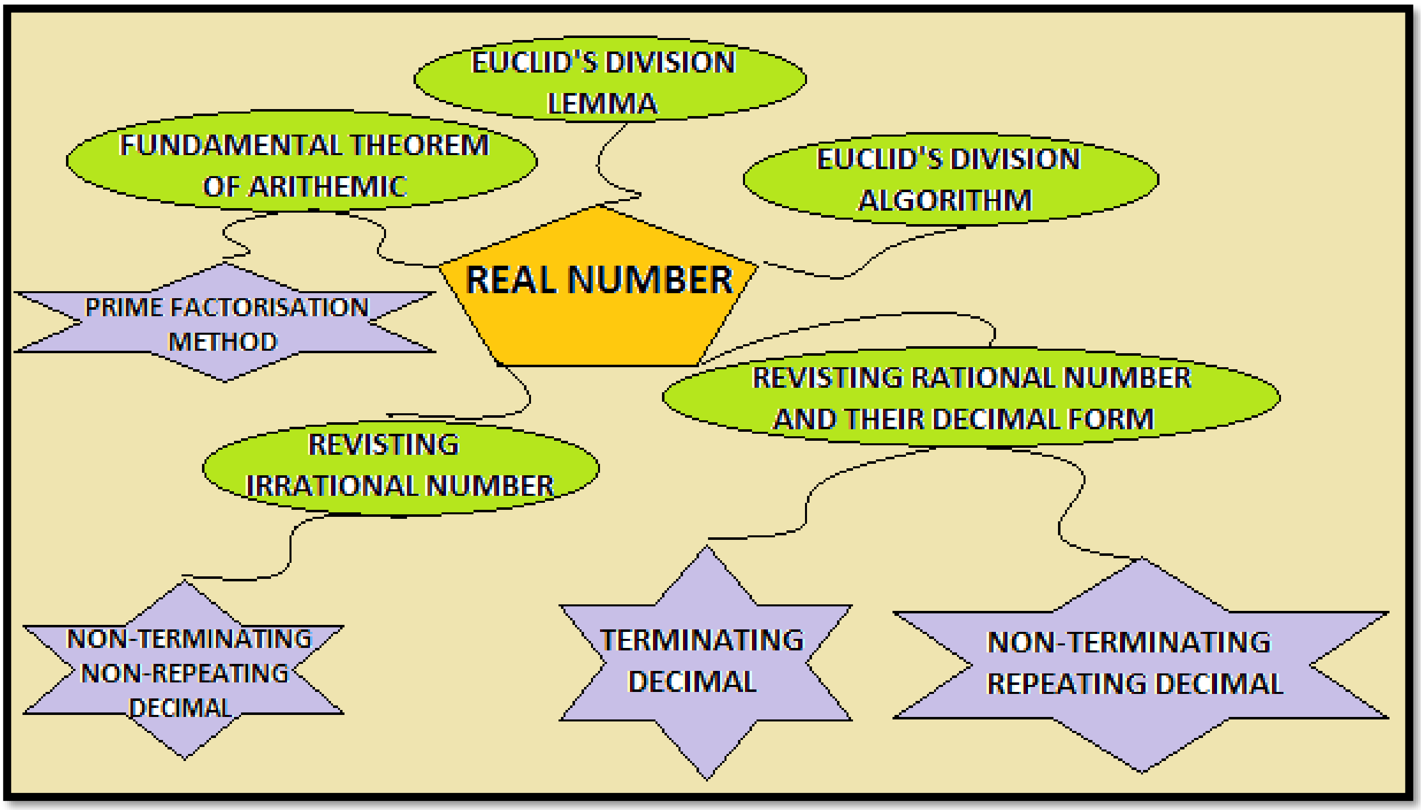
CLASS-X

TOPIC: REAL NUMBER + POLYNOMIAL

DAY-1

- **TEACHING MATERIAL**

CONCEPT MAP



Method of Finding LCM

Example: To find the Least Common Multiple (L.C.M) of 36 and 56.

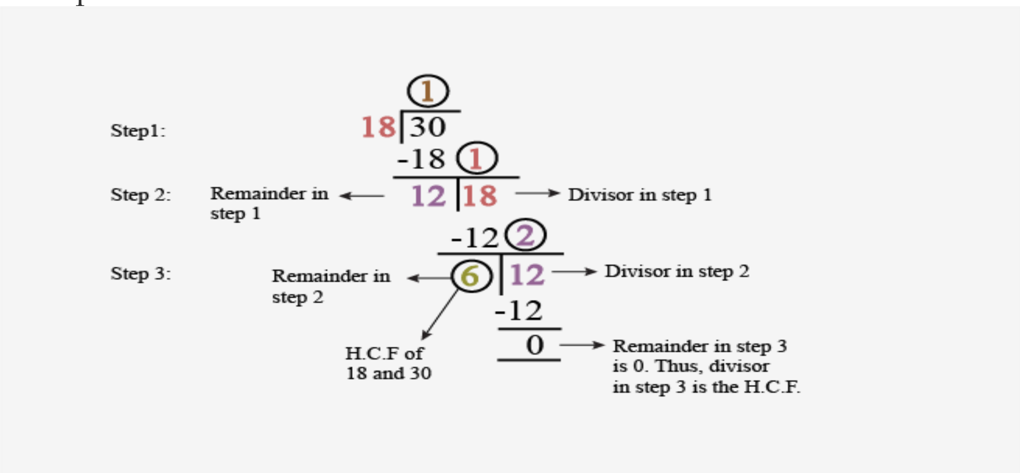
1. 36=2×2×3×3
   56=2×2×2×7
2. The common prime factors are 2×2
3. The uncommon prime factors are 3×3 for 36 and 2×7 for 56.
4. LCM of 36 and 56 = 2×2×3×3×2×7 which is 504

Method of Finding HCF

H.C.F can be found using two methods – Prime factorization and Euclid's division algorithm.

- Prime Factorization:
  - Given two numbers, we express both of them as products of their respective prime factors. Then, we select the prime factors that are common to both the numbers
  - Example – To find the H.C.F of 20 and 24
    20=2×2×5 and 24=2×2×2×3
  - The factor common to 20 and 24 is 2×2, which is 4, which in turn is the H.C.F of 20 and 24.

- Euclid's Division Algorithm:
  - It is the repeated use of Euclid's division lemma to find the H.C.F of two numbers.
  - Example: To find the HCF of 18 and 30



- The required HCF is 6.

Product of Two Numbers = HCF X LCM of the Two Numbers

- For any two positive integers a and b,
  a × b 0= H.C.F × L.C.M.
- Example – For 36 and 56, the H.C.F is 4 and the L.C.M is 504
  36×56=2016 , 4×504=2016
  Thus, 36×56=4×504
- The above relationship, however, doesn't hold true for 3 or more numbers

Applications of HCF & LCM in Real-World Problems

- L.C.M can be used to find the points of common occurrence. This could be the common ringing of bells that ring with different frequencies, the time at which two persons running at different speeds meet, and so on

- **VIDEO LINKS**

https://youtu.be/-s06Os761dU

Solve the following questions

1. Use Euclid's division algorithm to find the HCF of 105 and 120.
2. If the LCM (91, 26) =182, then find the HCF (91, 26).
3. Find the largest number which divide 615 and 963 leaving remainder 6 in each case.
4. Can two numbers have 18 as their HCF and 380 as their LCM ? Give reason.
5. Find the greatest number of 6 digits exactly divisible by 24, 15 and 36.

DAY-2

❖ **TEACHING MATERIAL**

Irrational Numbers

Any number that cannot be expressed in the form of p/q (where p and q are integers and q≠0.) is an irrational number. Examples √2, π, e and so on.

Number theory

- If a number p (a prime number) divides a2, then p divides a.

For example – 3 divides 62 i.e. 36, which implies that 3 divides 6.

- The sum or difference of a rational and an irrational number is irrational
- The product and quotient of a non-zero rational and irrational number are irrational.
- √p is irrational when 'p' is a prime. For example, 7 is a prime number and √7 is irrational. The above statement can be proved by the method of "Proof by contradiction".

Proof by Contradiction

In the method of contradiction, to check whether a statement is TRUE
(i) We assume that the given statement is TRUE.
(ii) We arrive at some result which contradicts our assumption, thereby proving the contrary.

Eg: Prove that √7 is irrational.

Assumption: √7 is rational.

Since it is rational √7 can be expressed as

√7 = $\frac{a}{b}$, where a and b are co-prime Integers, b ≠ 0.

On squaring $\frac{a2}{b^2}$ = 7

⇒ a2 = 7b2.

Hence, 7 divides a. Then, there exists a number c such that a=7c. Then, a2=49 c2. Hence, 7b2 = 49 c2 or b2 = 7c2.
Hence 7 divides b. Since 7 is a common factor for both a and b, it contradicts our assumption that a and b are coprime integers.
Hence, our initial assumption that √7 is rational is wrong. Therefore, √7 is irrational.

❖ VIDEO LINKS

https://byjus.com/maths/irrational-numbers/

https://youtu.be/mX91_3GQqLY

https://youtu.be/dutF-1K4HK8

Solve the following questions

1. Prove that $\sqrt{7}$ is an irrational number.
2. Prove that $2 + \sqrt{3}$ is an irrational number.

DAY-3

❖ TEACHING MATERIAL

Rational Numbers

Rational numbers are numbers that can be written in the form $\frac{p}{q}$, where p and q are integers and q≠0.
Examples – $\frac{1}{2}, \frac{4}{5}, \frac{-3}{4}$ and so on.

Terminating and non-terminating decimals

Terminating decimals are decimals that end at a certain point. Example: 0.2, 2.56 and so on.
Non-terminating decimals are decimals where the digits after the decimal point don't terminate. Example: 0.333333….., 0.13135235343…

Non-terminating decimals can be :
a) Recurring – a part of the decimal repeats indefinitely (0.142857142857….)
b) Non-recurring – no part of the decimal repeats indefinitely. Example: π = 3.1415926535…

Check if a given rational number is terminating or not

If $\frac{a}{b}$ is a rational number, then its decimal expansion would terminate if both of the following conditions are satisfied :
a) The H.C.F of a and b is 1.
b) b can be expressed as a prime factorization of 2 and 5 i.e. b=2m×5n where either m or n, or both can = 0.
If the prime factorization of b contains any number other than 2 or 5, then the decimal expansion of that number will be recurring

Example:

$\frac{1}{40}$ = 0.025 is a terminating decimal, as the H.C.F of 1 and 40 is 1, and the denominator (40) can be expressed as 23×51.

$\frac{1}{40}$= 0.428571 is a recurring decimal as the H.C.F of 3 and 7 is 1 and the denominator (7) is equal to 71

❖ **VIDEO LINKS**

https://youtu.be/m7xdPzGsktU

Solve the following questions

1. The decimal expansion of a real number is 23.123456. If it is expressed as a rational number in the form of $\frac{p}{q}$, write the prime factors of q
2. Without actual division, state where the rational number $\frac{183}{375}$ is a terminating decimal expansion or non- terminating decimal expansion.
3. Express 3.423423…….. in the form $\frac{p}{q}$, where p and q are integer and q≠ 0

DAY-4

❖ **TEACHING MATERIAL**

Algebraic Expressions

An algebraic expression is an expression made up of variables and constants along with mathematical operators.

An algebraic expression is a sum of terms, which are considered to be building blocks for expressions.

A term is a product of variables and constants. A term can be an algebraic expression in itself.
Examples of a term – 3 which is just a constant.
– 2x, which is the product of constant '2' and the variable 'x'
– 4xy, which is the product of the constant '4' and the variables 'x' and 'y'.
– 5 x 2y, which is the product of 5, x, x and y.

The constant in each term is referred to as the coefficient.

Example of an algebraic expression – 3x2y+4xy+5x+6 which is the sum of four terms – 3x2y, 4xy, 5x and 6

An algebraic expression can have any number of terms. The coefficient in each term can be any real number. There can be any number of variables in an algebraic expression. The exponent on the variables, however, must be rational numbers.

Polynomial

An algebraic expression can have exponents that are rational numbers. However, a polynomial is an algebraic expression in which the exponent on any variable is a whole number.

5x3 + 3x + 1 is an example of a polynomial. It is an algebraic expression as well

2x+3√x is an algebraic expression, but not a polynomial. – since the exponent on x is 1/2 which is not a whole number.

Degree of a Polynomial

For a polynomial in one variable – the highest exponent on the variable in a polynomial is the degree of the polynomial.

Example: The degree of the polynomial x2+2x+3 is 2, as the highest power of x in the given expression is x2.

TYPES OF POLYNOMIALS

Polynomials can be classified based on

a) Number of terms
b) Degree of the polynomial.

Types of polynomials based on the number of terms

a) Monomial – A polynomial with just one term. Example – 2x, 6x2, 9xy

b) Binomial – A polynomial with two terms. Example – 4x2+x, 5x+4

a) Trinomial – A polynomial with three terms. Example – x2+3x+4

Types of Polynomials based on Degree

Linear Polynomial

A polynomial whose degree is one is called a linear polynomial.
For example, 2x+1 is a linear polynomial.

Quadratic Polynomial

A polynomial of degree two is called a quadratic polynomial.
For example, 3x2+8x+5 is a quadratic polynomial.

Cubic Polynomial

A polynomial of degree three is called a cubic polynomial.
For example, 2x3+5x2+9x+15 is a cubic polynomial.

❖ **VIDEO LINKS**

https://youtu.be/SeejizAqSlY

DAY-5

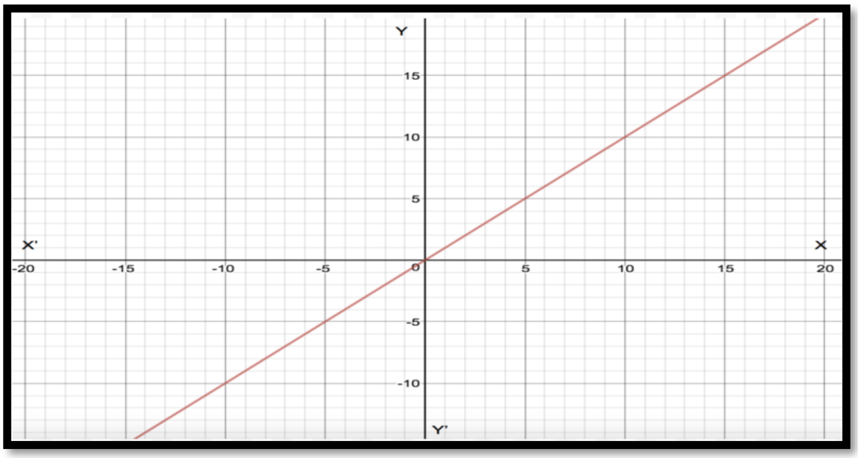
❖ **TEACHING MATERIAL**

Graphical Representations

Representing Equations on a Graph

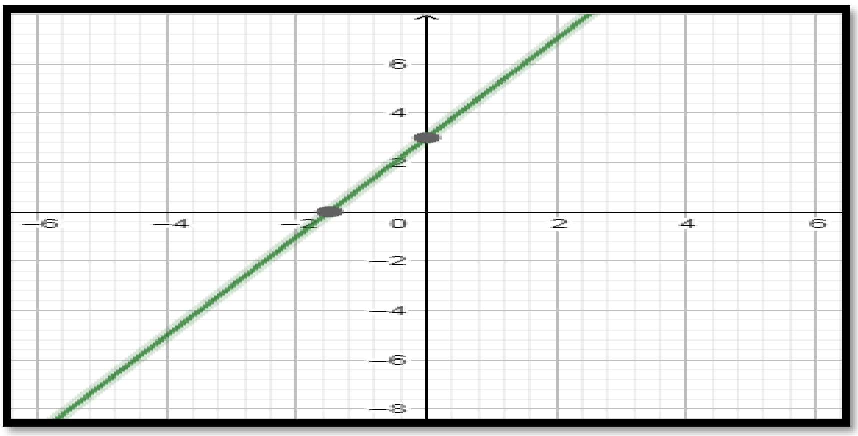
Any equation can be represented as a graph on the Cartesian plane, where each point on the graph represents the x and y coordinates of the point that satisfies the equation. An equation can be seen as a constraint placed on the x and y coordinates of a point, and any point that satisfies that constraint will lie on the curve

For example, the equation y = x, on a graph, will be a straight line that joins all the points which have their x coordinate equal to their y coordinate. Example – (1,1), (2,2) and so on.



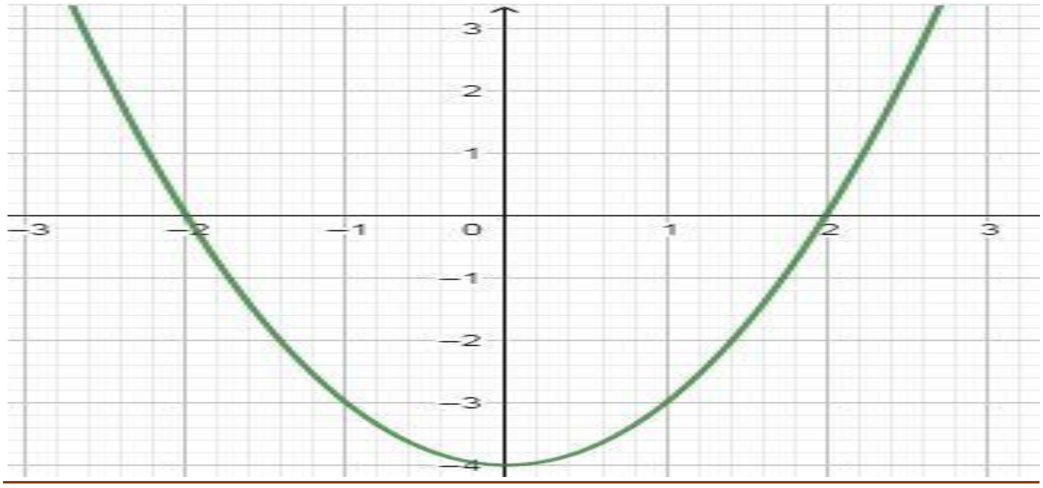
Visualization of a Polynomial

- Geometrical Representation of a Linear Polynomial
- The graph of a linear polynomial is a straight line. It cuts the X-axis at exactly one point.

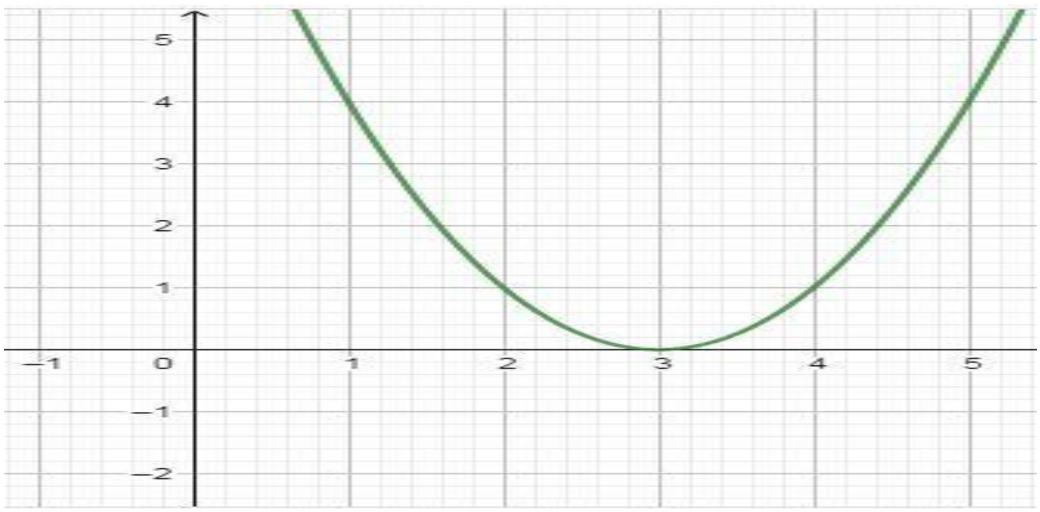


Geometrical Representation of a Quadratic Polynomial

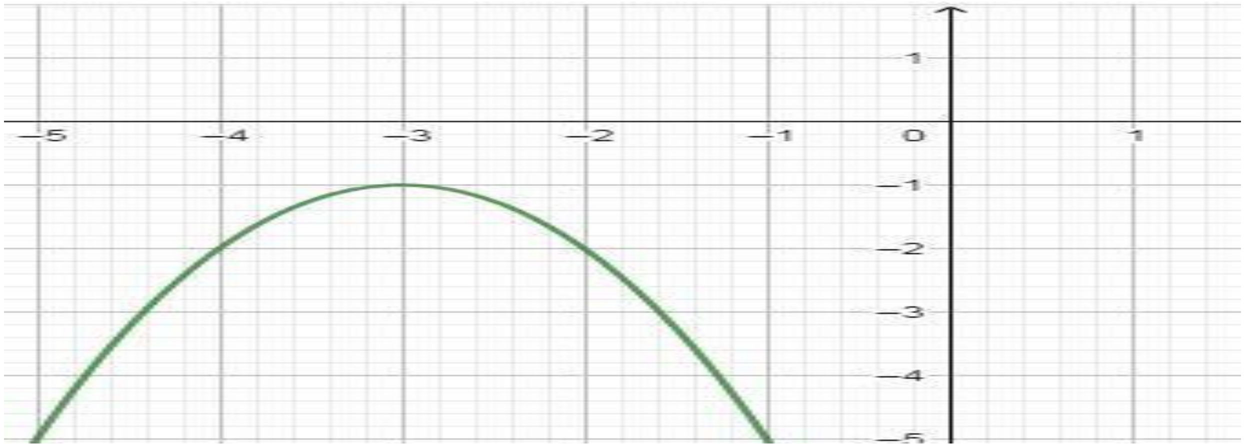
- The graph of a quadratic polynomial is a parabola.
- It looks like a U which either opens upwards or opens downwards depending on the value of a in ax2+bx+c.
- If a is positive then parabola opens upwards and if a is negative then it opens downwards.
- It can cut the x-axis at 0, 1 or two points.



Graph of a polynomial which cuts the x-axis in two distinct points (a>0)



Graph of a Quadratic polynomial which touches the x-axis at one point (a>0)

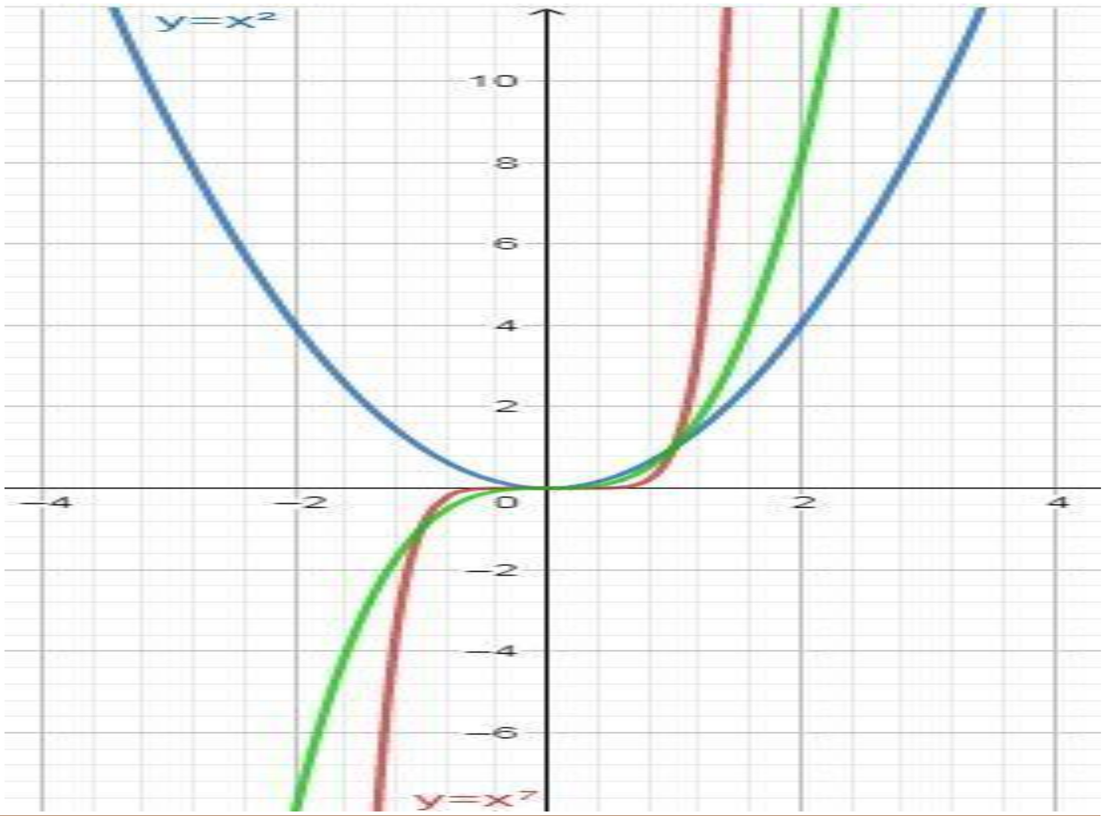


Graph of a Quadratic polynomial that doesn't touch the x-axis (a<0)

Graph of the polynomial xn

For a polynomial of the form y=xn where n is a whole number:

- as n increases, the graph becomes steeper or draws closer to the Y-axis.
- If n is odd, the graph lies in the first and third quadrants
- If n is even, the graph lies in the first and second quadrants.
- The graph of y=−xn is the reflection of the graph of y=xn on the x-axis



Graph of polynomials with different degrees.

Zeroes of a Polynomial

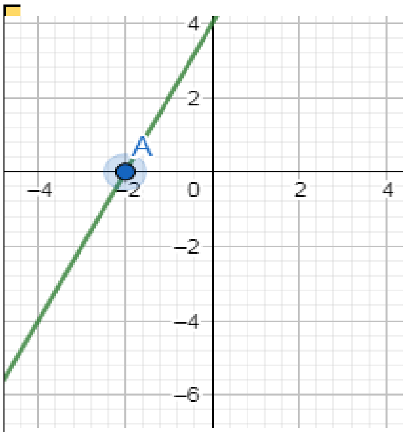
A zero of a polynomial p(x) is the value of x for which the value of p(x) is 0. If k is a zero of p(x), then p(k)=0.

For example, consider a polynomial $p(x)=x2-3x+2$.
When x=1, the value of p(x) will be equal to
$p(1)=12-3\times1+2$
$=1-3+2$
$=0$

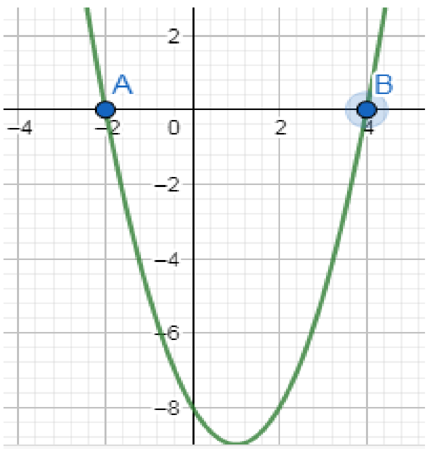
Since p(x)=0 at x=1, we say that 1 is a zero of the polynomial $x2-3x+2$

Geometrical Meaning of Zeros of a Polynomial

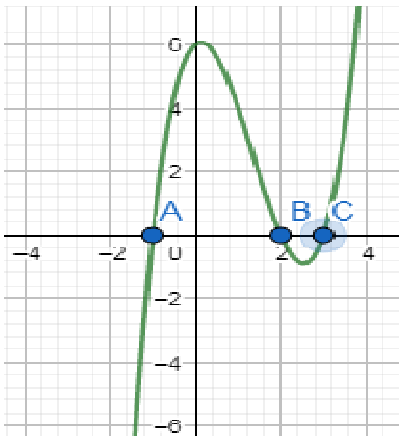
Geometrically, zeros of a polynomial are the points where its graph cuts the x-axis.



(i) One zero



(ii) Two zeros



(iii) Three zeros

Here A, B and C correspond to the zeros of the polynomial represented

❖ **VIDEO LINKS**

https://youtube.be/dutF-1K4HK8

https://youtu.be/Dr9RuGk-BfE

Home Assignment

1. Write whether every positive integer can be of the form 4q + 2, where q is an integer. Justify your answer

2. Find the least number that is divisible by all the numbers from 1 to 10(both inclusive)

3. If two positive integers p and q can be expressed as p = and q = b; a, b being prime numbers, Find the LCM (p, q)

4. Using Euclid's division algorithm, find which of the following pairs of numbers are co-prime: (i) 231, 396 (ii) 847, 2160

5. If n is an odd integer, then show that – 1 is divisible by 8.

6. Using Euclid's division algorithm, find the largest number that divides 1251, 9377 and 15628 leaving remainders 1, 2 and 3, respectively.

7. Prove that one and only one out of n, n + 2 and n + 4 is divisible by 3, where n is any positive integer.

8. A rational number in its decimal expansion is 327.7081. What can you say about the prime factors of q

9. Without actually performing the long division, find if $\frac{987}{10500}$ will have terminating or non-terminating (repeating) decimal expansion.

10. The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.