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Sarvada Sarva shreshth
HOLISTIC DEVELOPMENT

ONLINE TEACHING MATERIAL

MATHEMATICS

SESSION-2020-21

CLASS-XII

TOPIC: RELATIONS AND FUNCTIONS

DAY-1

Sub topic : Relations

1. RELATION:

A relation R from a set A to a set B is a subset of A x B , where A and B are any two non-empty sets i.e., R is a relation fro A to B if $R \subseteq A \times B$. If (x , y) $\in$R, then, we write xRy (read as x is related to y under the relation R) and if (x,y) $\notin$R, then we write x $\not R$ y(read as x is not related to y under the relation R).

2. DOMAIN AND RANGE OF A RELATION:

(a) Domain of R is the set of all first coordinates of elements of R and it is denoted By Dom(R).

(b) Range of R is the set of all second coordinates of R and it is denoted by Range(R)

A relation R on set A means, the relation from A to A i.e., $R \subseteq A \times A$.

3. SOME STANDARD TYPES OF RELATIONS:

Let A be a non-empty set. Then, a relation R on set A is said to be

(a) Reflexive: If (x , x) $\in$R for each element x $\in$A, i.e., if x R x for each element x $\in$A.

(b) Symmetric: If (x, y) $\in$R => (y, x) $\in$R for all x, y $\in$A.

(c) Transitive: If (x , y) $\in$ R and (y , z) $\in$R=> (x , z) $\in$R for all x , y, z $\in$A, i.e., if xRy and yRz => xRz.

NCERT MATERIAL:

http://ncert.nic.in/ncerts/l/leep201.pdf

Video link

https://youtu.be/oNCIgGg-EhU

Day 2

EQUIVALENCE RELATION :

Any relation R on a set A is said to be an equivalence relation if R is reflexive, symmetric and transitive.

NCERT MATERIAL:

https://www.google.com/url?sa=t&source=web&rct=j&url=http://ncert.nic.in/ncerts/l/keep202.pdf&ved=2ahUKEwil1fjQrsLoAhXXfH0KHVJUD0YQFjAAegQIBBAB&usg=AOvVaw1u5mZk3zl6DDO51bNedgGF

Video link :

https://youtu.be/8rkdpUIgTgE

Day 3

5. FUNCTION:

Let X and Y be two non-empty sets. Then, a rule f which associates to each element $x \in X$, a unique element, denoted by f(x) of Y, is called a function from X to Y and written as f: X -> Y where , f(x) is called image of x and x is called the pre-image of f(x) and the set Y is called the co-domain of f and $f(X) = \{f(x) : x \in X\}$ is called the range of f.

6. TYPES OF FUNCTION:

(i) One-one function (injective function): A function f: X->Y is defined to be one-one if the image of distinct element of X under rule f are distinct, i.e., for every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies that $x_1=x_2$.

(ii) Onto function (Surjective function) : A function f: X->Y is said to be onto function if each element of Y is the image of some element of x i.e., for every $y \in Y$ there exists some $x \in X$ such that y = f(x). Thus f is onto if range of f = co-domain of f.

(iii) Many one- function: A function f: X->Y is said to be a many-one function if two or more elements of set X have the same image in Y. i.e., f: X->Y is said to be a many-one function if there exists a, b ∈ X such that $a \neq b$ but f(a) = f(b).

Video link :

https://youtu.be/E5K2oJyo03U

7. COMPOSITION OF FUNCTIONS:

Let f: A-> B and g: B-> C be two functions. Then, the composition of f on g, denoted by gof is defined as the function gof : A->C given by gof(x)= g(f(x)), for all x ∈ A. Clearly, dom (g o f) = dom (f). Also, gof is defined only when range(f) $\subseteq$ dom(g).

Video link :

https://youtu.be/wUNWjd4bMmw

8. INVERTIBLE FUNCTIONS :

For f : A-> B, if there exists a function g : B-> A such that gof= I_A and fog = I_B, where I_A and I_B are identity functions, then f is called an invertible function, and g is called the inverse of f and it is written as f^{-1} = g.

.Video link :
https://youtu.be/mPQCHmOxGlY

https://youtu.be/W84lObmOp8M

https://www.khanacademy.org/math/in-in-grade-12-ncert/in-in-relations-functions/copy-of-math3-inverse-func-intro/v/introduction-to-function-inverses?modal=1

FEW MORE POINTS

Types of Relations

A relation in set A is a subset of A **×** A. Thus, A **×** A is two extreme relations.

Empty Relation

If no element of A is related to any element of A, i.e. R = φ ⊂ A **×A,** then the relation in a set is called empty relation.

Universal Relation

If each element of A is related to every element of A, i.e. R = A **×** A, then the relation is said to be universal relation.

A relation R in a set A is called-

Reflexive- if (a,a) ∈ R, for every a ∈ A.

Symmetric- if (a_1,a_2) ∈ R implies that (a_2,a_1) ∈ R , for all a_1,a_2∈ A.

Transitive- if (a_1,a_2) ∈ R and (a_2,a_3) ∈ R implies that (a_1,a_3) ∈ R for all a_1,a_2,a_3 ∈ A.

Equivalence Relation- A relation in a set A is equivalence relation if R is reflexive, symmetric and transitive.

Composition of Functions and Invertible Function

Let f : A → B and g : B → C be two functions. Then the composition of f and g, denoted by **gof**, is defined as the function gof : A → C given by;

gof (x) = g(f (x)), ∀ x ∈ A

Binary Operations

A binary operation ∗ on a set A is a function ∗ : A × A → A. We denote ∗ (a, b) by a ∗ b.

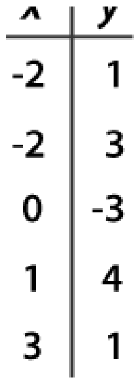
Example: Show that subtraction and division are not binary operations on R.

Solution: R × R → R, given by (a, b) → a – b, is not binary operation, as the image of (3, 5) under '–' is 3 – 5 = – 2 ∉ R.

Similarly, ÷: R × R → R, given by (a, b) → a ÷ b is not a binary operation, as the image of (3, 5) under ÷ is 3 ÷ 5 = 3 5 ∉ R.

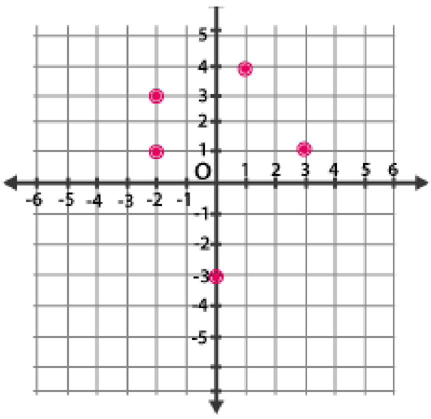
Relation Representation

There are other ways too to write the relation, apart from set notation such as through tables, plotting it on XY-axis or through mapping diagram.

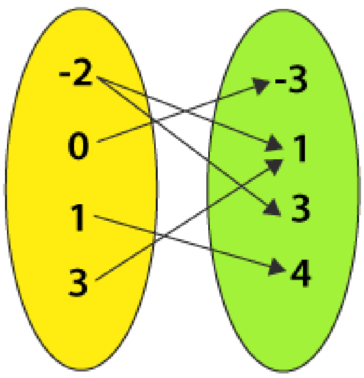


x	y
-2	1
-2	3
0	-3
1	4
3	1

Relation in table



Relation in graph



Relation in mapping diagran

Types of Relations

Different types of relations are as follows:

- Empty Relations
- Universal Relations
- Identity Relations
- Inverse Relations
- Reflexive Relations
- Symmetric Relations
- Transitive Relations

Let us discuss all the types one by one.

Empty Relation

When there's no element of set X is related or mapped to any element of X, then the relation R in A is an empty relation also called as void. I.e R= ∅.

For example,

if there are 100 mangoes in the fruit basket. There's no possibility of finding a relation R of getting any apple in the basket. So, R is Void as it has 100 mangoes and no apples.

Universal relation

R is a relation in a set, let's say A is a universal Relation because, in this full relation, every element of A is related to every element of A. i.e R = A × A.

It's a full relation as every element of Set A is in Set B.

Identity Relation

If every element of set A is related to itself only, it is called Identity relation.

I={(A, A), ∈ a}.

For Example,

When we throw a dice, the outcome we get is 36. I.e (1, 1) (1, 2), (1, 3)…..(6, 6). From these, if we consider the relation(1, 1), (2, 2), (3, 3) (4, 4) (5, 5) (6, 6), it is an identity relation.

Inverse Relation

If R is a relation from set A to set B i.e R ∈ A X B. The relation R^{-1} = {(b,a):(a,b) ∈ R}.
For Example,

If you throw two dice if R = {(1, 2) (2, 3)}, R^{-1}= {(2, 1) (3, 2)}. Here the domain is the Range R^{-1} and vice versa.

Reflexive Relation

A relation is a reflexive relation If every element of set A maps to itself. I.e for every a ∈ A,(a, a) ∈ R.

Symmetric Relation

A symmetric relation is a relation R on a set A if (a,b) ∈ R then (b, a) ∈ R, for all a &b ∈ A.

Transitive Relation

If (a,b) ∈ R, (b,c) ∈ R, then (a,c) ∈ R, for all a,b,c ∈ A and this relation in set A is transitive.

Equivalence Relation

If and only if a relation is reflexive, symmetric and transitive, it is called an equivalence relation.

PPT link :

https://www.slideshare.net/indupsthakur/relations-functionspps

https://www.slideshare.net/Dreams4school/relations-and-functions-42999445

Concepts and Formulae Key Concepts

http://im.rediff.com/getahead/2010/feb/19relations-functions-trig.pdf

LAB ACTIVITIES

http://ncert.nic.in/ncerts/l/lelm501.pdf

Determining if a function is invertible

https://youtu.be/dqCTAHHza10

Introduction to the inverse of a function

https://youtu.be/-eAzhBZgq28

Proof: Invertibility implies a unique solution to f(x)=y

https://youtu.be/7GEUgRcnfVE

Surjective (onto) and injective (one-to-one) functions

https://youtu.be/xKNX8BUWR0g

1 MARK QUESTIONS

1. Check whether the relation R in the set {1,2,3} given by R = {(1, 2), (2,1)} is transitive.
2. Find the number of all onto functions from the set { 1, 2, 3,□;n} to itself.
3. If f: R-> R is given by f(x)= (3 □ $x^3)^{1/3}$, determine f(f(x)).
4. Find fog(x), if f(x) = $|x|$ and g(x) = $|5x-2|$.
5. If f(x) = x + 7 and g(x) = x -7, x∈ R, find fog(7).
6. If f(x) is an invertible function, find the inverse of f(x) = $\frac{3x-2}{5}$.
7. Find fog; if f(x)= $8x^3$ and g(x) = $x^{1/3}$.
8. If f(x) = x^2 + 4 then find $f^{-1}(x)$.
9. If A = {3 , 5, 7} and B = { 2, 4, 9} and R is a relation from A to B given by □is less than□, write R as a set of ordered pairs.

10. If f : R- {0} -> R □ {0} is defined as f(x) = 3/x, find $f^{-1}(x)$.
11. If f(x) = x2 + 1, g(x)= 1/(x+1), find gof(5).
12. Let A = {1,2,3}, B = {4,5,6,7} and let f = {(1,4), (2,5), (3,6), (3,6)} be a function from A to B. State whether f is one one or not.

4 MARK QUESTIONS

1. Let L be the set of all lines in xy plane and R be the relation in L defined as R= { (L_1 ,L_2): L_1 is parallel to L_2}. Show that R is an equivalence relation. Find the set of all lines related to the line y = 2x + 4.
2. Let T be the set of all triangles in a plane and R be the relation in T defined as R= { (T_1 ,T_2): T_1 is congruent to T_2}. Show that R is an equivalence relation.
3. If the function f: [1, ∞) -> [1, ∞) defined by f(x) = $2^{x(x-1)}$ is invertible, find $f^{-1}(x)$.
4. Find the value of parameter α for which the function f(x) = 1 + αx, α≠0 is the inverse of itself.

7. Show that the function f : R -> R given by f(x)= $x^3 + x$ is a bijection.
8. If f(x) = $\sqrt{x}$ (x≥0) and g(x) = x^2 -1 are two real functions, find fog and g o f. Is fog= g o f?
9. Let f : N-> R be a function defined as f(x)= $4x^2 + 12x + 15$. Show that f:N->Range(f) is symmetric.
10. If f(x) = $\log\frac{1+x}{1-x}$, prove that f($\frac{2x}{1+x^2}$) = 2f(x).
11. Show that f: R − {-1} ->R − {-1} given by f(x) = $\frac{x}{x+1}$ is invertible. Also find f^{-1}.
12. Prove that the function f: N->N , defined by f(x)= x^2 + x +1 is one-one but not onto.
13. Show that the function f : N-> N given by f(n) = n − $(-1)^n$ for all n$\in$N is a bijection.
14. Consider the identity function I_N : N-> N defined as (I_N + I_N)(x) = I_N(x) + I_N(x) = x + x = 2x is not onto.
15. Consider the function f: [0 , π/2]-> R given by f(x) = sin x and g:[0, π/2]-> R given by g(x) = cos x. Show that f and g are one-one, but f + g is not one-one.
16. If the function f: R -> R be given by f(x) = x^2 + 2 and g: R-> R be given by g(x) = $\frac{x}{x-1}$. Find fog and gof.
17. If f: R- {7/5} -> R − {3/5} be defined as f(x) = $\frac{3x+4}{5x-7}$ and g : R- {3/5} -> R- {7/5} be defined as g(x) = $\frac{7x+4}{5x-3}$. Show that g o f= I_A and fog = I_B, where B = R- {3/5} and A = R − {7/5}.
18. If f : R -> R be given by f(x) = $\sin^2 x + \sin^2(x + \frac{\pi}{3}) + \cos x \cos(x + \frac{\pi}{3})$ for all x$\in$R, and g: R -> R be such that g(5/4) = 1, then prove that g o f: R->R is a constant function.
19. Let f: R-> R be a function given by f(x)= ax + b for all x$\in$R. Find the constants a and b such that f o f = I_R
20. Let f : A-> A be a function such that fof = f. Show that f is onto if and only if f is one-one. Describe f in this case.
21. Let $f: R \to R$ be defined as f(x) = 10x + 7. Find the function $g: R \to R$ such that gof =fog= I_R

6 MARK QUESTIONS

1. Show that the relation R on the set A of points in a plane given by R= { (P, Q) : distance of the point P from the origin is same as the distance of the point Q from the origin}, is an equivalence relation. Further, show that the set of all points related to a point P≠ (0, 0) is the circle passing through P with origin as centre.
2. Show that the relation R on the set A = { x $\in$Z: 0≤x≤12}, given by R = { (a, b) : $|a-b|$ is a multiple of 4} is an equivalence relation. Also, find the equivalence class of the element 1.
3. On the set M = A(x) = { $\begin{vmatrix} x & x \\ x & x \end{vmatrix} : x \in R$ } of 2 x 2 matrices, find the identity element for the multiplication of matrices as a binary operation. Also find the inverse of an element of M.
4. Let N denote the set of all natural numbers and R be the relation on N x N defined by (a, b) R (c, d) ⇔ad(b + c). Check whether R is an equivalence relation on N x N.