BISHOP SCOTT BOYS' SCHOOL

(Affiliated to CBSE, New Delhi) Affiliation No.: 330726, School Campus: Chainpur, Jaganpura, By-Pass, Patna 804453.
Phone Number: 7061717782, 9798903550. ,
Web: www.bishopscottboysschool.com Email: info@bishopscottboysschool.com

ONLINE TEACHING MATERIAL

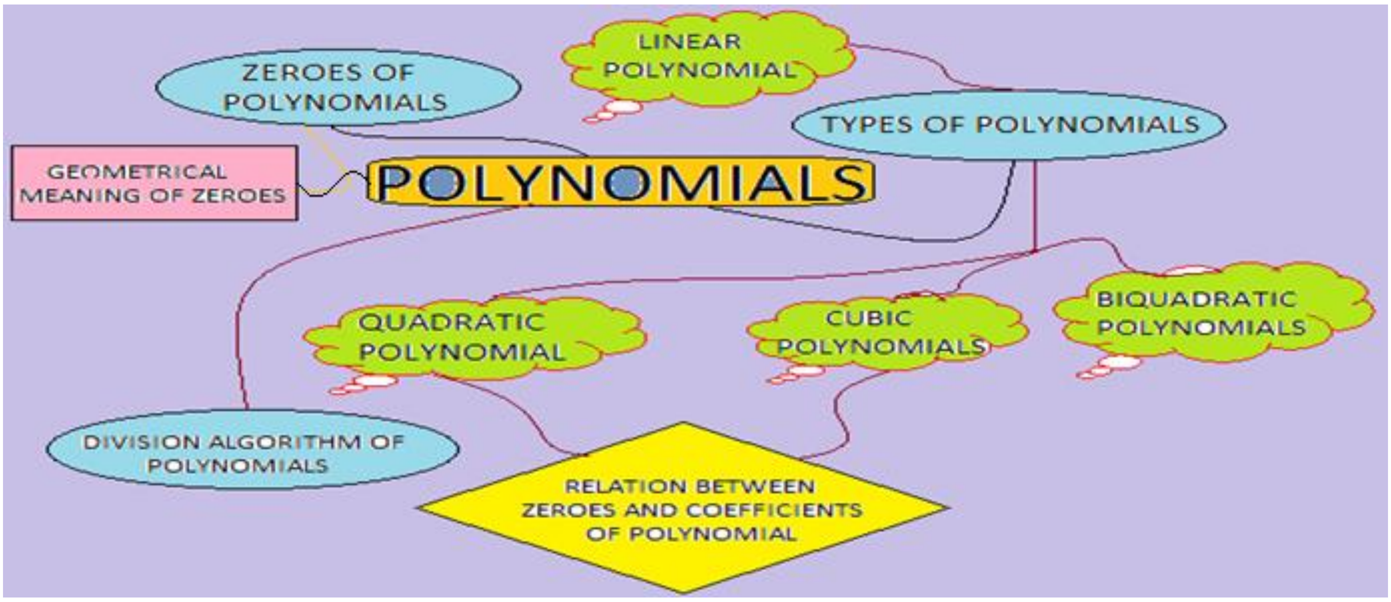
SUBJECT- MATHS

SESSION-2020-21

CLASS-_10_

TOPIC- POLYNOMIALS

CONCEPT MAP



Day – 1

Zeroes of polynomial

When we calculated zeroes of polynomial p(x) = x – 1, we equated it to 0, x – 1 = 0 or, x = 1. Hence, we say that p(x) = 0 is the Polynomial Equation. 1 is the Root of the Polynomial equation p(x) = 0. OR 1 is the Zero of the Polynomial equation p(x) = x – 1 = 0.

Now, let's look at a constant polynomial '5'. You can write this as $5x^0$. What is the Root of this constant polynomial? The answer is a Non-zero constant polynomial has no zero. Also, every real number is a zero of the Zero Polynomial.

Let's look at the following linear polynomial to understand the calculation of the roots or 'zeroes of polynomial': p(x) = ax + b … where a ≠ 0. To find a zero, we must equate the polynomial to 0. [p(x) = 0]. Hence, ax + b = 0 … where a ≠ 0. So, ax = – b or, x = – b/a. Therefore, '- b/a' is the only zero of p(x) = ax + b. We can also say that a linear polynomial has only one zero.

NOTE

- A zero of a polynomial need not be 0.
- 0 may be a zero of a polynomial.
- Every linear polynomial has one and only one zero.
- A polynomial can have more than one zeroes.
- In general a polynomial of degree n has at most n zeros. Thus

  ⇒A linear polynomial has one zero,

  ⇒A quadratic polynomial has at most two zeros.

  ⇒A cubic polynomial has at most 3 zeros.

***Solved Examples*:**

Verify whether the following are zeroes of polynomials indicated against them.

(1). p(x) = 3x + 1, x = – 1/3

(2). p(x) = 5x – π, x = 4/5

(3)p(x) = (x + 1) (x – 2), x = – 1, 2

(4)p(x) = $3x^2$ – 1, x = – 1/√3 , 2/√3

Solution:

(1) p(x) = 3x + 1, x = – 1/3
p(-1/3) = 3(-1/3) + 1 = – 1 + 1 = 0.
Hence, x = -1/3 is a zero of polynomial 3x + 1.

(2) p(x) = 5x – π, x = 4/5
p(4/5) = 5(4/5) – π = 4 – π ≠ 0.
Hence, x = 4/5 is not a zero of polynomial 5x – π.

(3) p(x) = (x + 1) (x – 2), x = – 1, 2
p(-1) = (– 1 + 1)(- 1 – 2) = (0)(- 3) = 0.
Hence, x = – 1 is a zero of polynomial (x + 1) (x – 2).
p(2) = (2 + 1)(2 – 2) = (3)(0) = 0.
Hence, x = 2 is a zero of polynomial (x + 1) (x – 2).

(4) p(x) = $3x^2$ – 1, x = – 1/√3 , 2/√3
p(- 1/√3) = 3(- 1/√3)^2 – 1 = 3 (1/3) – 1 = 1 – 1 = 0.
Hence, x = – 1/√3 is a zero of polynomial $3x^2$ – 1.

$p(2/\sqrt{3}) = 3(2/\sqrt{3}) – 1 = 3(4/3) – 1 = 4 – 1 = 3 \neq 0$.
Hence, $x = 2/\sqrt{3}$ is not a zero of polynomial $3x^2 – 1$.

Related Link

https://www.youtube.com/watch?v=QXOLozXkAvs

Day – 2

Quadratic Polynomials

A polynomial which is in the form of $ax^2 + bx + c$, $a \neq 0$(where a, b and c are real numbers) is known a quadratic polynomial.
Degree of quadratic polynomial is 2, so every quadratic polynomial has two zeroes.

The factorization of Quadratic Polynomials

Quadratic polynomials can be factorized by splitting the middle term.

For example, consider the polynomial $2x^2-5x+3$

Splitting the middle term.

The middle term in the polynomial $2x^2-5x+3$ is -5. This must be expressed as a sum of two terms such that the product of their coefficients is equal to the product of 2 and 3 (coefficient of x^2 and the constant term)

-5 can be expressed as $(-2)+(-3)$, as $-2\times-3=6=2\times3$

Thus, $2x^2-5x+3 = 2x^2-2x-3x+3$

Now, identify the common factors in individual groups

$2x^2 - 2x - 3x + 3 = 2x (x-1) - 3 (x-1)$

Taking $(x-1)$ as the common factor, this can be expressed as

$2x (x-1) - 3(x-1) = (x-1) (2x-3)$

Relationship between Zeroes and Coefficients of a Quadratic Polynomial:

In general, if α and β are the zeroes of the quadratic polynomial $p(x) = ax^2 + bx + c$, $a \neq 0$, then we know that $(x - \alpha)$ and $(x - \beta)$ are the factors of p(x).

$$\text{Sum of roots } \alpha + \beta = \frac{-b}{a} \Rightarrow \alpha + \beta = \frac{-(\text{coefficient of } x)}{\text{coefficient of } x^2}$$

$$\text{Product of roots} = \alpha\beta = \frac{c}{a} \Rightarrow \alpha\beta = \frac{\text{constant term}}{\text{coefficient of } x^2}$$

NOTE: - If α, β are roots of a quadratic polynomial p(x), then $p(x) = x^2 - (\alpha + \beta) x + \alpha \beta$
$\Rightarrow p(x) = x^2 - (\text{sum of roots}) x + \text{product of roots}$

Solved Examples:

1. Find the zeroes of the quadratic polynomial $x^2 + 7x + 10$, and verify the relationship between the zeroes and the coefficients.

Sol. On finding the factors of $x^2 + 7x + 10$, we get, $x^2 + 7x + 10 = (x + 2)(x + 5)$

Thus, value of $x^2 + 7x + 10$ is zero for $(x+2) = 0$ or $(x+5) = 0$. Or in other words, for $x = -2$ or $x = -5$.

Hence, zeros of $x^2 + 7x + 10$ are -2 and -5.

Now, sum of zeros = -2 + (-5) = -7 = -7/1 = -(Coefficient of x)/(Coefficient of x^2). Similarly, product of zeros = (-2) x (-5) = 10 = 10/1 = (Constant term)/ (Coefficient of x^2).

2. Find the zeroes of the quadratic polynomial $t^2 - 15$, and verify the relationship between the zeroes and the coefficients.

Sol. On finding the factors of $t^2 - 15$, we get, $t^2 - 15 = (t + \sqrt{15})(t - \sqrt{15})$

Thus, value of $t^2 - 15$ is zero for $(t + \sqrt{15}) = 0$ or $(t - \sqrt{15}) = 0$.

Or in other words, for $t = \sqrt{15}$ or $t = -\sqrt{15}$.

Hence, zeros of $t^2 - 15$ are $\sqrt{15}$ and $-\sqrt{15}$.

Now, sum of zeros = $\sqrt{15} + (-\sqrt{15}) = 0 = -0/1$ = -(Coefficient of t)/(Coefficient of t^2). Similarly, product of zeros = $(\sqrt{15}) \times (-\sqrt{15}) = -15 = -15/1$ = (Constant term)/ (Coefficient of t^2).

3. Find a quadratic polynomial for the given numbers as the sum and product of its zeroes respectively 4, 1.

Sol. Let the quadratic polynomial be $ax^2 + bx + c$.

Given, $\alpha + \beta = 4 = 4/1 = -b/a$.

$\alpha \beta = 1 = 1/1 = c/a$.

Thus, a = 1, b = -4 and c = 1.

Therefore, the quadratic polynomial is $x^2 - 4x + 1$.

4. Find a quadratic polynomial whose zeroes are $5 + \sqrt{2}$ and $5 - \sqrt{2}$.

Sol. Let α, β are zeroes of quadratic polynomial p(x).

$\therefore p(x) = x2 - (\alpha + \beta)x + \alpha\beta$

Here, $\alpha = 5 + \sqrt{2}$, $\beta = 5 - \sqrt{2}$

$\therefore \alpha + \beta = 5 + \sqrt{2} + 5 - \sqrt{2} = 10$

and $\alpha\beta = (5 + \sqrt{2})(5 - \sqrt{2})$

$= 25 - 4 = 21$

$\therefore p(x) = x^2 - 10x + 21$

5. Find a quadratic polynomial, the sum of whose zeroes is 0 and one zero is 5.

Sol. Let zeroes are α and β.

$\Rightarrow \alpha + \beta$ = Sum of zeroes

$\Rightarrow \alpha + \beta = 0 \Rightarrow 5 + \beta = 0 \Rightarrow \beta = -5$

Now product of zeroes = $\alpha\beta = 5 \times (-5) = -25$

Let polynomial $p(x) = ax^2 + bx + c$

$\therefore \alpha + \beta = 0 = \frac{-b}{a}; \alpha\beta = \frac{c}{a} = -25$

$\therefore a = 1, b = 0, c = -25$

Hence, $p(x) = x^2 - 25$

Related video link :- **https://youtu.be/ITvifxhHDkk**

Day – 3

Cubic Polynomials

A polynomial which is in the form of $ax^3 + bx^2 + cx+d$, $a \neq 0$(where a, b, c and d are real numbers) is known a Cubic polynomial.
Degree of cubic polynomial is 3, so every quadratic polynomial has three zeroes.

Relationship between Zeroes and Coefficients of a cubic Polynomial:

If α, β and γ are zeroes of $p(x) = ax^3 + bx^2 + cx + d$

Then, sum of roots $= \alpha + \beta + \gamma = \frac{-b}{a} = \frac{-(\text{coefficient of } x^2)}{\text{coefficient of } x^3}$

Sum of product of roots taken, two at a time

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

Product of roots $= \frac{-d}{a} \Rightarrow \alpha\beta\gamma = \frac{-(\text{constant term})}{\text{coefficient of } x^3}$

NOTE:- If α, β and γ are zeroes of a cubic polynomial p(x),
Then, $p(x) = x^3 – (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma) x – (\alpha\beta\gamma)$
$\Rightarrow p(x) = x^3$ – (sum of zeroes) x^2 + (sum of product of zeroes / roots taken two at a time) x – (Product of zeroes)

Example:- Find a cubic polynomial with the sum, sum of the product of its zeroes taken two at a time, and product of its zeroes are 5, –6 and –20 respectively.

Sol. Let $p(x) = ax^3 + bx^2 +cx + d$
If α, β and γ are its zeroes of p(x),

Here, Sum of zeroes = $\alpha + \beta + \gamma = 5$
Sum of the products of zeroes taken two at a time = $\alpha\beta + \alpha\gamma + \beta\gamma = – 6$

Product of zeroes = $\alpha\beta\gamma = – 20$
We know that,
Polynomial $p(x) = x^3 – (\alpha + \beta + \gamma) x^2 + (\alpha\beta + \beta\gamma + \alpha\gamma) x – (\alpha\beta\gamma)$
$= x^3 – 5x^2 – 6x + 20$

Related Link: - https://youtu.be/YanxPXXvCPQ

Day – 4

Division Algorithm for a Polynomial

If p(x) and g(x) are any two polynomials with g(x) ≠ 0, then we can find polynomials q(x) and r(x) such that
p(x) = q(x) × g(x) + r(x), where r(x) = 0 or degree of r(x) < degree of g(x).

To divide one polynomial by another, follow the steps given below.

Step 1: arrange the terms of the dividend and the divisor in the decreasing order of their degrees.

Step 2: To obtain the first term of the quotient, divide the highest degree term of the dividend by the highest degree term of the divisor Then carry out the division process.

Step 3: The remainder from the previous division becomes the dividend for the next step. Repeat this process until the degree of the remainder is less than the degree of the divisor.

Solved Examples

1. Divide $4x^3 + 2x^2 + 5x - 6$ by $2x^2 + 3x + 1$.

Sol.

$$\begin{array}{r|l} & 2x - 2 \\ \hline 2x^2 + 3x + 1 & 4x^3 + 2x^2 + 5x - 6 \\ & 4x^3 + 6x^2 + 2x \\ & - \quad - \quad - \\ \hline & -4x^2 + 3x - 6 \\ & -4x^2 - 6x - 2 \\ & + \quad + \quad + \\ \hline & 9x - 4 \end{array}$$

∴ Quotient is $2x - 2$ and remainder is $9x - 4$.

2. Find all zeroes of polynomial $4x^4 - 20x^3 + 23x^2 + 5x - 6$ if two of its zeroes are 2 and 3.

Sol. Since two zeroes are 2 and 3.
∴ $(x - 2)(x - 3)$ is a factor of given polynomial.
⇒ $x^2 - 5x + 6$ is a factor of given polynomial.
Now

$$\begin{array}{r|l} & 4x^2 - 1 \\ \hline x^2 - 5x + 6 & 4x^4 - 20x^3 + 23x^2 + 5x - 6 \\ & 4x^4 - 20x^3 + 24x^2 \\ & - \quad + \quad - \\ \hline & -x^2 + 5x - 6 \\ & -x^2 + 5x - 6 \\ & + \quad - \quad + \\ \hline & 0 \end{array}$$

∴ $4x^4 - 20x^3 + 23x^2 + 5x - 6$
$= (x^2 - 5x + 6)(4x^2 - 1)$
$= (x - 2)(x - 3)(2x - 1)(2x + 1)$
∴ Zeroes of the given polynomial are
$2, 3, \frac{1}{2}, \frac{-1}{2}$.

Related Link: - https://youtu.be/RAfWldSNG5k

Day – 5

Learn the following identities:-

Algebraic Identities

1. $(a + b)^2 = a^2 + 2ab + b^2$
2. $(a - b)^2 = a^2 - 2ab + b^2$
3. $(x + a)(x + b) = x^2 + (a + b)x + ab$
4. $a^2 - b^2 = (a + b)(a - b)$
5. $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
6. $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
7. $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$
8. $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

Related Link

https://youtu.be/nJ2DQTLbSVM

PPT LINK

https://www.slideshare.net/Nihask/polynomials-class-10

Assignments

Solve the following

1. Find the zeroes of the following polynomials and verify the relation between the zeroes and the coefficients :-
   (a) $x^2 - 2x - 8$ (b) $2\sqrt{3}x^2 - 5x + \sqrt{3}$
2. Find the quadratic polynomial whose zeroes are $\frac{2}{3}$ and $\frac{-1}{4}$. Verify the relationship between the coefficients and the zeroes of the polynomial.
3. If $x = \frac{2}{3}$ and $x = -3$ are the roots of the quadratic equation $ax^2+7x+b = 0$, then find the value of a and b.
4. If α and β be the zeroes of the polynomial $2x^2 + 5x + k$ such that $\alpha^2 + \beta^2 + \alpha\beta = \frac{21}{4}$ then find the value of K .
5. Find cubic polynomials whose zeroes are 2,-3 and 4.
6. Find the quotient and the remainder when $f(x) = x^4 - 3x^2 + 4x + 5$ is divided by $g(x) = x^2 + 1 - x$.
7. Verify division algorithm for the polynomials $f(x) = 8 + 20x + x^2 - 6x^3$ and $g(x) = 2 + 5x - 3x^2$.
8. If 3 and -3 are two zeroes of the polynomial $x^4 + x^3 -- 11x^2 - 9x + 18$, find all the zeroes of the given polynomial .
9. Find all the zeroes of the polynomial $2x^4-11x^3+7x^2+13x-7$,it being given that two of its zeroes are $(3 + \sqrt{2})$ and $(3 - \sqrt{2})$.
10. If the zeroes of the polynomial $f(x) = x^3 - 3x^2 + x + 1$ are $(\mathbf{a} - \mathbf{b})$, $\mathbf{a}$ and $(\mathbf{a} + \mathbf{b})$, then find **a** and **b**.