

STUDY COURSE MATERIAL

MATHEMATICS

SESSION-2020-21

CLASS-7th

TOPIC: TRIANGLES AND IT'S PROPERTIES

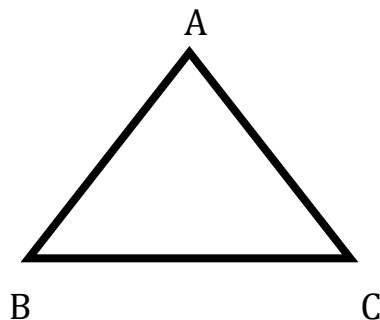
DAY-1

NCERT MATERIAL

- <https://play.google.com/store/apps/details?id=com.ncert>

TEACHING MATERIAL

Triangle – A closed figure bounded by three line segments is known as triangle.



Let A, B, C be three non collinear points. Then the figure formed by the three line segments AB, BC, CA is called a triangle.

It is denoted as ΔABC

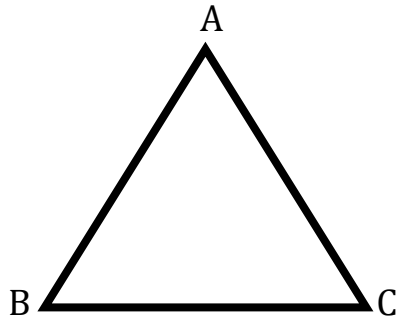
So, ΔABC has

- Three sides, namely AB, BC, CA
- Three angles namely $\angle ABC$, $\angle BCA$, $\angle CAB$, simply $\angle A$, $\angle B$, $\angle C$
- Three vertices A, B and C

VARIOUS TYPES OF TRIANGLES

NAMING TRIANGLE BY CONSIDERING THEIR ANGLES

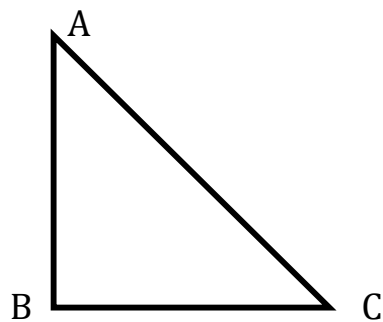
1) **Acute Triangle** – A triangle each of whose angle measures less than 90° is called an acute angle.



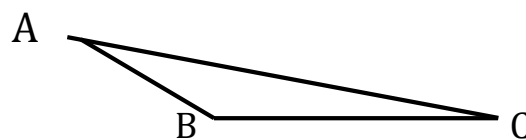
Here, $\angle A = 55^\circ$, $\angle B = 65^\circ$, $\angle C = 60^\circ$

2) **Right Triangle** – A triangle one of whose angle measures 90° is called right triangle.

Here, $\angle B = 90^\circ$. So, ΔABC is a right angled triangle.



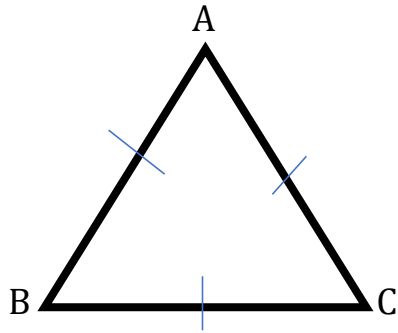
3. **Obtuse Triangle**- A triangle having one angle greater than 90° and less than 180° degrees is known as obtuse angled triangle.



Here $\angle B = 110^\circ$, so ΔABC is an obtuse angle triangle.

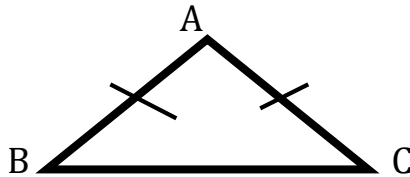
NAMING TRIANGLE BY CONSIDERING THE LENGTH OF THEIR SIDES

1. **Equilateral triangle** - A triangle having all side equal is called an equilateral triangle.



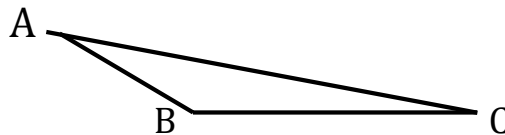
Here, $AB=BC=AC$, So, ΔABC is an equilateral.

2. **Isosceles Triangle** - A triangle having two sides equal is called an isosceles triangle.



Here, $AB=AC$, So, ΔABC is an isosceles triangle.

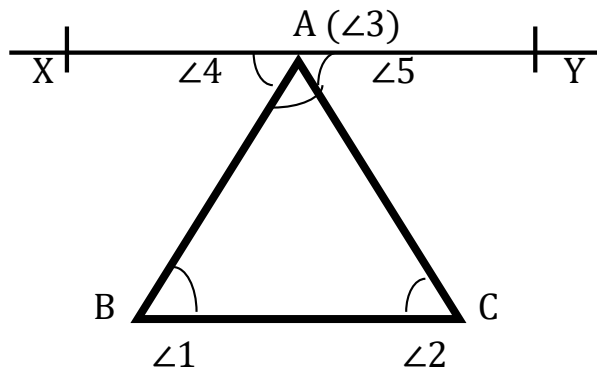
3. **Scalene Triangle** - A triangle having all sides of different length is called a scalene triangle.



Here, $AB \neq AC \neq BC$, So, ΔABC is a scalene triangle.

Angle sum property of a triangle

Theorem: Prove that the sum of interior angles of a triangle is 180° .



Given: ΔABC

To Prove: $\angle A + \angle B + \angle C = 180^\circ$

Construction: Draw $XY \parallel BC$

Proof: Now, $XY \parallel BC$ and the transversal AB cuts them.

$$\angle 1 = \angle 4 \text{ (Alternate interior angles)----- (i)}$$

Again $XY \parallel BC$ and the transversal AC cuts them.

$$\angle 2 = \angle 5 \text{ (Alternate interior angles)----- (ii)}$$

$$\text{Now, } \angle 4 + \angle 3 + \angle 5 = 180^\circ \text{ (XAY is a line)}$$

$$\angle 1 + \angle 3 + \angle 2 = 180^\circ \text{ (Using i \& ii)}$$

$$\text{Hence, } \angle A + \angle B + \angle C = 180^\circ \text{ (Proved).}$$

Important result

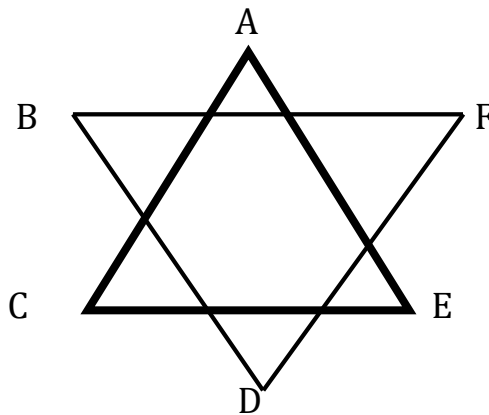
1. Each angle of an equilateral triangle measures 60° .
2. The angles opposite to equal sides of an isosceles triangle are equal.
3. A triangle cannot have more than one right angle.
4. A triangle cannot have more than one obtuse angle.
5. In a right triangle the sum of two acute angles is 90° .

VIDEO LINKS: <https://youtu.be/bh5qImMB8GI>

PRACTICE QUESTIONS

1. In a ΔABC $\angle A = 35^\circ$ and $\angle B = 65^\circ$, find the measure of $\angle C$.

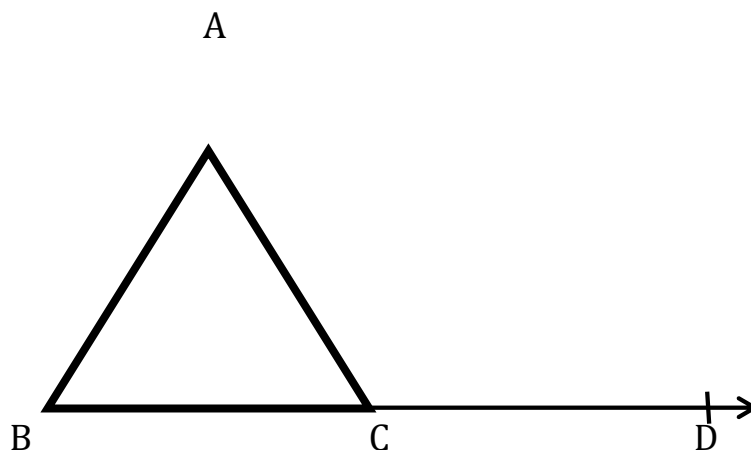
2. In a ΔABC if $3\angle A = 4\angle B = 6\angle C$. Calculate $\angle A, \angle B$ and $\angle C$.
3. If one angle of a triangle is equal to the sum of the other two, show that the triangle is right angled
4. Find the angles of triangle which are in ratio 3:4:5.
5. The adjoining figure has been obtained by using two triangles. Prove that $\angle A + \angle B + \angle C + \angle D + \angle E + \angle F = 360^\circ$.



DAY -2

Exterior and interior opposite angles

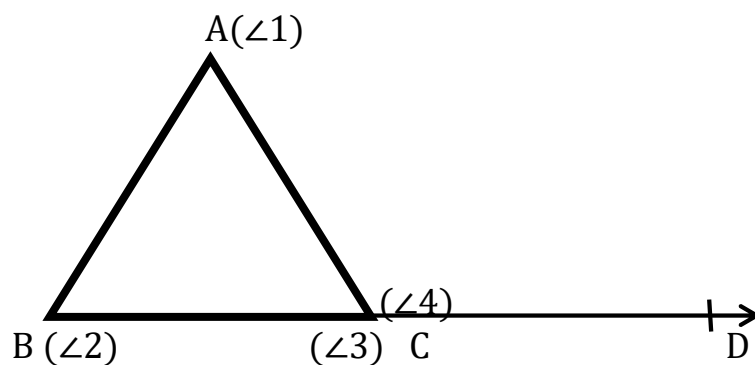
Let the side BC of a triangle ABC is produced to D then, $\angle ACD$ is called an exterior angle. Also $\angle BAC$ and $\angle ABC$ are called the interior opposite angles.



Here, $\angle ACD$ is exterior angle to ΔABC when side BC produced to D.

Exterior angle property of a Triangle

Theorem: If each side of a triangle is produced then the exterior angle so formed is equal to the sum of the two interior opposite angles.



Proof : Let the side BC of $\triangle ABC$ is produced to D, forming exterior angle $\angle ACD$. We know that the sum of angle of a triangle is 180°

$$\angle 1 + \angle 3 + \angle 2 = 180^\circ \text{ --- (i)}$$

$$\text{But, } \angle 3 + \angle 4 = 180^\circ \text{ --- (ii) (Linear Pair)}$$

From i & ii we get,

$$\angle 1 + \angle 3 + \angle 2 = \angle 3 + \angle 4$$

Hence , $\angle 1 + \angle 2 = \angle 4$ Proved.

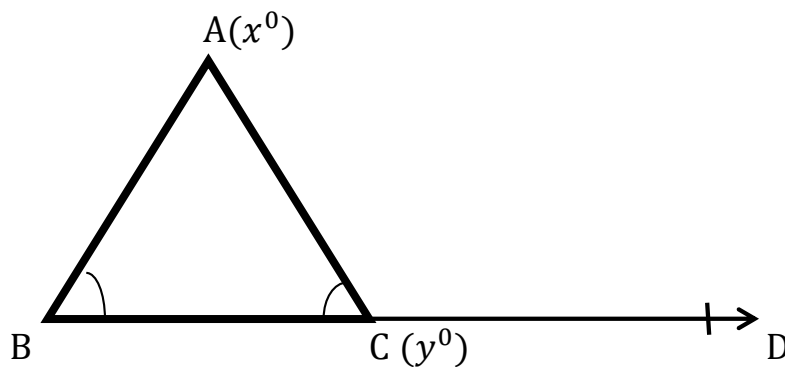
Important points

- Exterior angle is greater than each one of the interior opposite angle.
- When all the sides of a triangle extend in a order (clockwise or anticlockwise) then the sum of exterior angles so formed is equal to 360° .

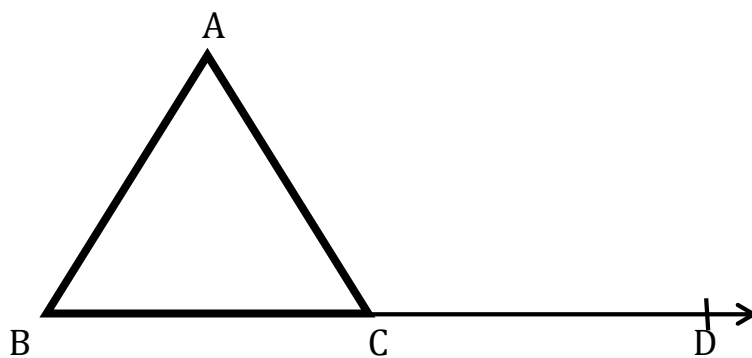
VIDEO LINKS: <https://youtu.be/EZ6d0lRQDBo>

PRACTICE QUESTIONS

1. In the given figure, find the value of x and y . If $\angle B = 55^\circ$, $\angle A = x$, $\angle C = y$, $\angle ACD = 125^\circ$

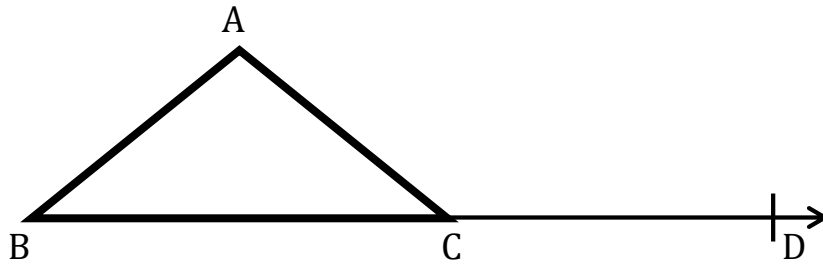


2. In the given figure given alongside, find the value of x and y . If $\angle B = 68^\circ$, $\angle A = x^\circ$, $\angle C = y^\circ$, $\angle ACD = 130^\circ$

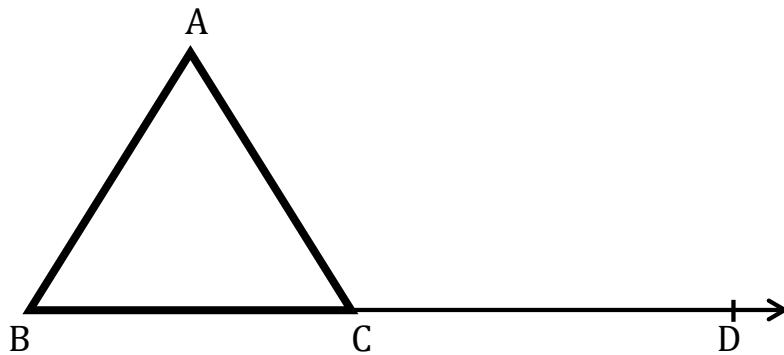


3. An exterior angle of a triangle measures 110° and its interior opposite angle are in the ratio 2:3. Find the angles of the triangle.

4. In the given figure alongside, $x : y$ equal to $2 : 3$ and $\angle ACD$ equals to 130° . Find the value of x, y and z . If $\angle A = y \angle B = x \angle C = z$.



5. In the given figure given below, find the measure of $\angle ACD$. If $\angle B = 45^\circ, \angle A = 75^\circ$.

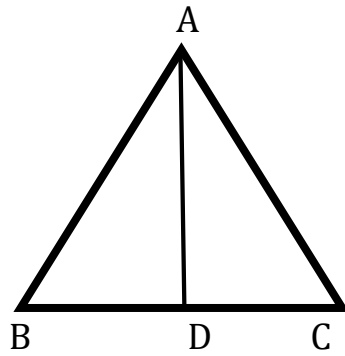


DAY - 3

TEACHING MATERIAL

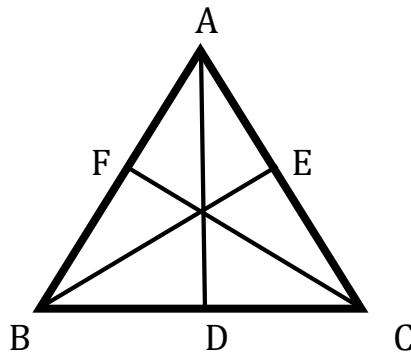
Some Term Related To Triangle

1. **Median**- A line segment which connects midpoint of a side to its opposite vertex is called median.



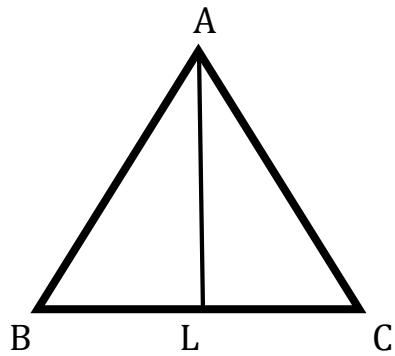
In the above figure, D is midpoint of BC.
So, line segment AD join vertex A to midpoint of side BC that is D.
Hence, AD is known as median.

Consider the following figure of ΔABC .



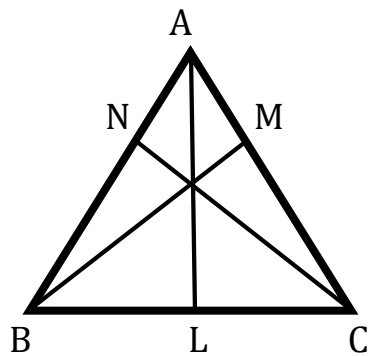
In the figure D, E, F are respectively the midpoint of sides BC, CA and AB of ΔABC

1. AD is the median, corresponding to side BC.
 2. BE if the medium corresponding to side CA.
 3. CF if the median, corresponding to the side AB.
-
2. **Altitude** - Altitude of a triangle is the perpendicular drawn from the vertex of the triangle to the opposite side. Also, known as the height of the triangle, the altitude makes a right angle triangle with the base.



In the above figure, line segment AL is an Altitude to base BC of triangle.

Consider the following figure of ΔABC



In the above figure,
AL is \perp to BC; BM \perp to CA and CN is \perp to AB

1. AL is the altitude, corresponding to base BC.
2. BM is the altitude corresponding to base CA.
3. CN is the altitude corresponding to base AB.

IMPORTANT RESULTS

1. The median of a triangle are concurrent.
2. **Centroid** - The point of intersection of all the three medians of a triangle is called its centroid.
3. The altitudes of a triangle are concurrent.
4. **Orthocentre** - The point of intersection of all the three altitudes of a triangle is called its orthocentre,

VIDEO LINKS:

- <https://youtu.be/i7F9Q8bqPVM>
- <https://youtu.be/ZIEAYFEgCH4>

PRACTICE QUESTIONS.

1. How many medians can a triangle have?
2. How many altitudes can a triangle have?
3. Draw a rough sketch for the following
 - i. In $\triangle ABC$, BE is a median.
 - ii. In $\triangle PQR$, PQ and PR are altitudes of triangle.
 - iii. In $\triangle XYZ$, YL is altitude in the exterior of the triangle.

DAY - 4

TEACHING MATERIAL

TRIANGLE INEQUALITY

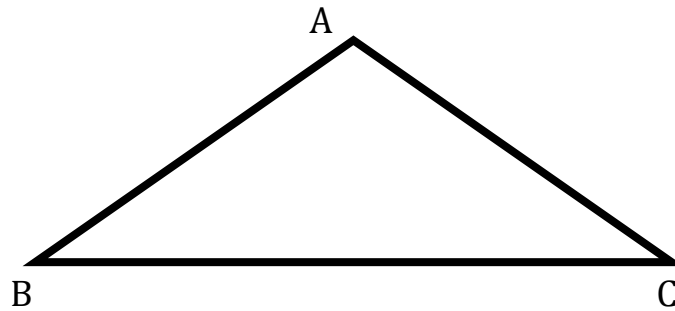
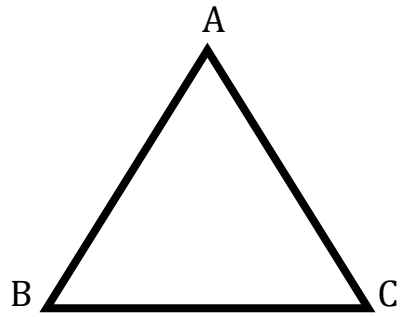
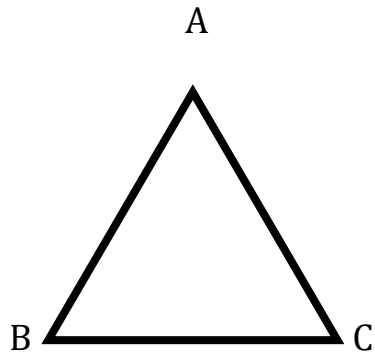
1. The sum of any two sides of a triangle is greater than the third side.
2. The difference of any two sides of a triangle is less than the third side.

Experiment

Draw three triangle T_1, T_2, T_3

Label each one as ABC

let $BC = a$, $AC = b$ and $AB = c$



Measure the length former a, b, c and tabulate observation

Observation table

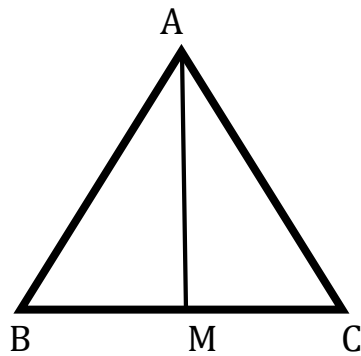
Triangle	Sides of ΔABC			computation					
	a	b	c	a+b	b+c	a+c	a+b-c	b+c-a	c+a+-b
T ₁									
T ₂									
T ₃									

RELATED EXERCISE

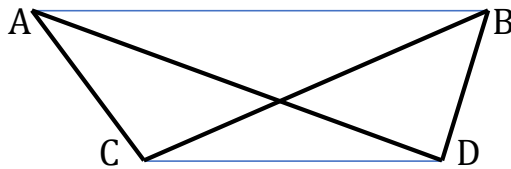
1. It is possible to draw a triangle, the length of sides are given below:

- i. 1 cm, 1 cm, 1 cm
- ii. 2 cm, 3 cm, 4 cm
- iii. 7 cm, 8 cm, 5 cm
- iv. 6 CM, 7 cm, 14 cm

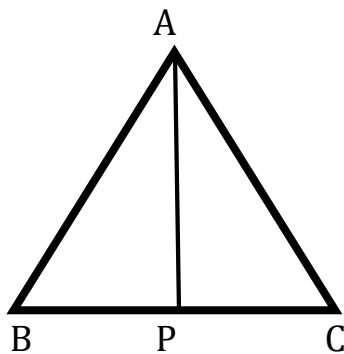
2. AM is a median of ΔABC prove that $(AB + BC + CA) > 2 AM$.



3. ABCD is a quadrilateral. Prove that $(AB + BC + CD + DA) > (AC + BD)$



4. In the given figure, P is a point on the side BC of ΔABC . Prove that $(AB + BC + AC) > 2AP$

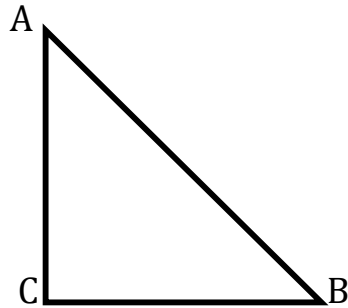


5. If O is a point in the exterior of ΔABC , show that $2(OA + OB + OC) > (AB + BC + CA)$

TEACHING MATERIAL

PYTHAGORAS' THEOREM

Statement: "In a right triangle the square of the hypotenuse is equal to the sum of the square of its remaining two sides."



Thus, In a right ΔABC in which angle $C = 90^\circ$ we have,
 $AB^2 = BC^2 + AC^2$

Thus, If $AB=c$, $BC=a$ and $AC=b$, we have:
 $c^2 = a^2 + b^2$

EXPERIMENT:

Draw any three right triangle say T_1, T_2, T_3

Label each one of them as ΔABC with $\angle C$ as right angle

In each case, measure the sides a , b and hypotenuse c of the triangle.

Compute a^2, b^2 and c^2 and tabulate the observation as under,

Observation Table

Right Triangle	Measurement			computation				
	a	b	c	a^2	b^2	a^2+b^2	c^2	$c^2-(a^2+b^2)$
T_1								
T_2								
T_3								

You will find that in each case,

$$c^2 - (a^2 + b^2) = 0$$

$$\text{Hence, } c^2 = a^2 + b^2$$

IMPORTANT RESULT

- In a right triangle, the hypotenuse is the longest side.
- The perpendicular line segment is the shortest distance of a point from a given line.

VIDEO LINKS :

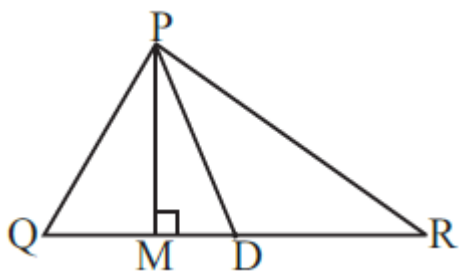
- <https://youtu.be/BiagrTl2y4o>
- <http://youtu.be/YompsDIEdtc>

PRACTICE QUESTION

1. Find the length of the hypotenuse of a right triangle. The other two sides of which measure 9 cm and 12 cm.
2. The hypotenuse of a right triangle is 26 cm long. If one of the remaining two sides is 10 cm long, find the length of the other side.
3. A 5 m long ladder when set against the wall of a house reaches a height of 4.8m. How far is the foot of the ladder from the wall?
4. A 15 metre long ladder is placed against a wall to reach a window 12 metre above the ground, find the distance of the foot of the ladder from the wall.

SOLVED QUESTIONS FOR REFERENCE

1. In $\triangle PQR$, D is the mid-point of \overline{QR} .



- (i) \overline{PM} is .

Solution:-

Altitude

An altitude has one end point at a vertex of the triangle and other on the line containing the opposite side.

(ii) PD is .

Solution:-

Median

A median connects a vertex of a triangle to the mid-point of the opposite side.

(iii) Is $QM = MR$?

Solution:-

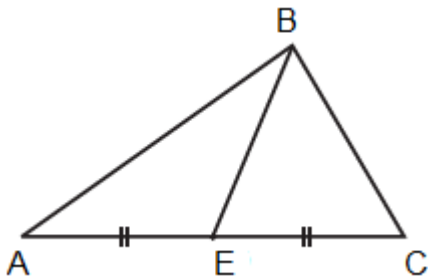
No, $QM \neq MR$ because, D is the mid-point of QR.

2. Draw rough sketches for the following:

(a) In $\triangle ABC$, BE is a median.

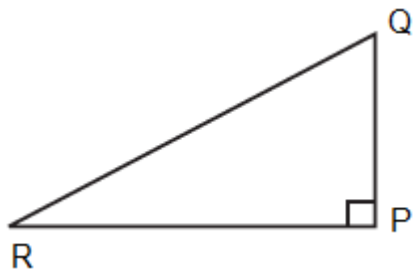
Solution:-

A median connects a vertex of a triangle to the mid-point of the opposite side.



(b) In $\triangle PQR$, PQ and PR are altitudes of the triangle.

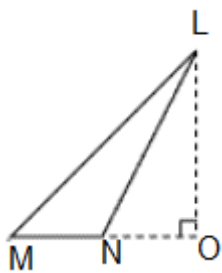
Solution:-



An altitude has one end point at a vertex of the triangle and other on the line containing the opposite side.

(c) In $\triangle XYZ$, YL is an altitude in the exterior of the triangle.

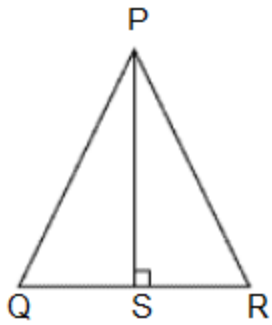
Solution:-



In the figure we may observe that for $\triangle LMN$, LO is an altitude drawn exteriorly to side LN which is extended up to point L.

3. Verify by drawing a diagram if the median and altitude of an isosceles triangle can be same.

Solution:-

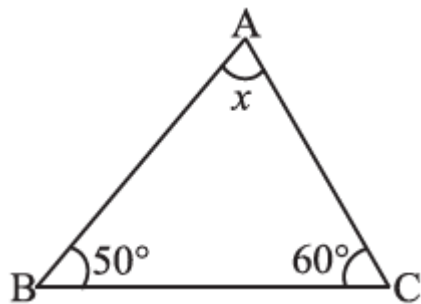


Draw a Line segment $PS \perp BC$. It is an altitude for this triangle. Here we observe that length of QS and SR is also same. So PS is also a median of this triangle.

SET-2

1. Find the value of the unknown x in the following diagrams:

(i)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180° .

Then,

$$= \angle BAC + \angle ABC + \angle BCA = 180^\circ$$

$$= x + 50^\circ + 60^\circ = 180^\circ$$

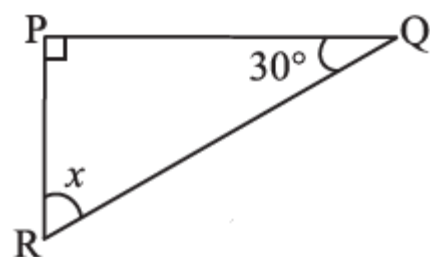
$$= x + 110^\circ = 180^\circ$$

By transposing 110° from LHS to RHS it becomes $- 110^\circ$

$$= x = 180^\circ - 110^\circ$$

$$= x = 70^\circ$$

(ii)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180° .

The given triangle is a right angled triangle. So the $\angle QPR$ is 90° .

Then,

$$= \angle QPR + \angle PQR + \angle PRQ = 180^\circ$$

$$= 90^\circ + 30^\circ + x = 180^\circ$$

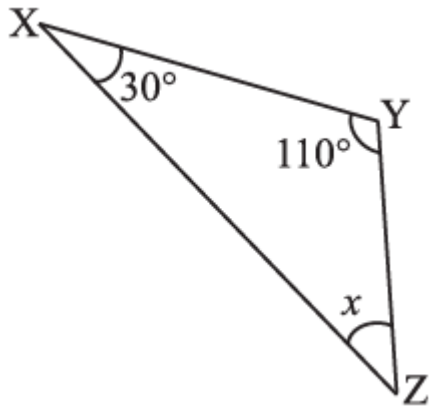
$$= 120^\circ + x = 180^\circ$$

By transposing 120° from LHS to RHS it becomes $- 120^\circ$

$$= x = 180^\circ - 120^\circ$$

$$= x = 60^\circ$$

(iii)



Solution:-

We know that,

The sum of all the interior angles of a triangle is 180° .

Then,

$$= \angle XYZ + \angle YXZ + \angle XZY = 180^\circ$$

$$= 110^\circ + 30^\circ + x = 180^\circ$$

$$= 140^\circ + x = 180^\circ$$

By transposing 140° from LHS to RHS it becomes $- 140^\circ$

$$= x = 180^\circ - 140^\circ$$

$$= x = 40^\circ$$

